First-Order Filters

Secton 14.4

First-order Switched-capacitor Filter

Active RC Prototype

Switched-capacitor Equivalent

Fig. 14.17

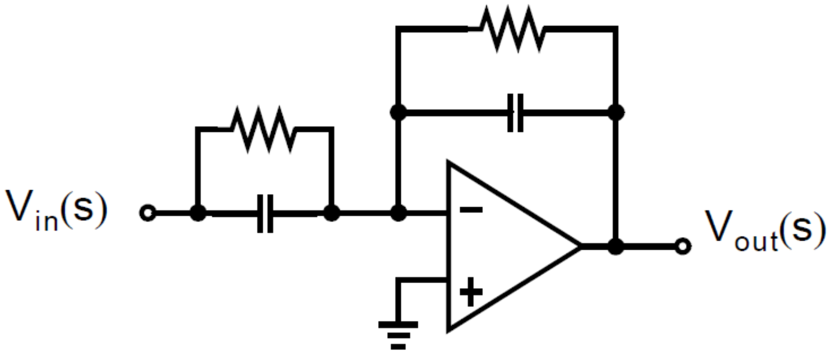
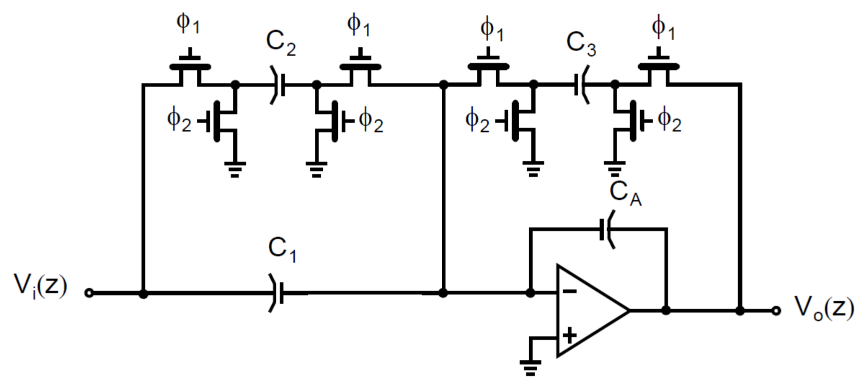


Fig. 14.18(a)

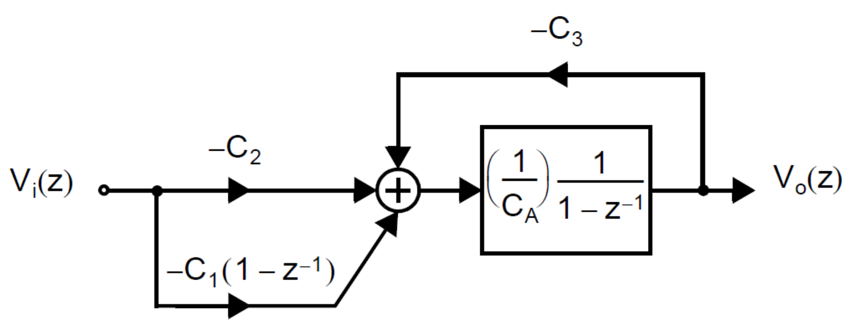


Beginning with conventional continuous-time active RC filters, and replacing each resistor with a switched-capacitor results in a discrete-time filter whose frequency response closely approximates that of the original continuous-time filter at frequencies far below the sampling frequency

First-order Switched-capacitor Filter

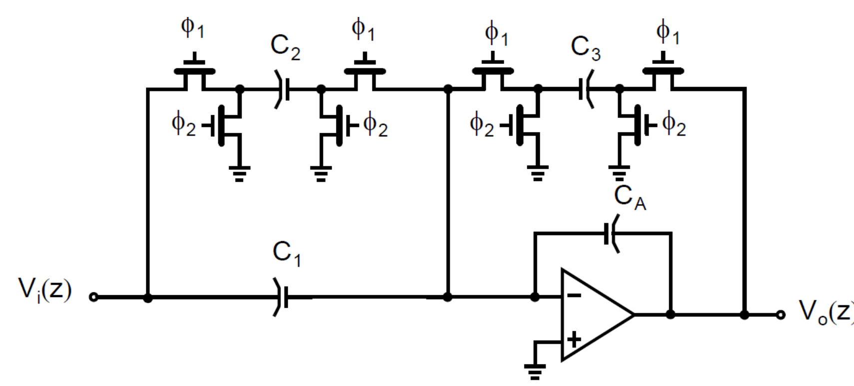
Signal Flow Graph

Fig. 14.18(b)



Switched-capacitor Filter

Fig. 14.18(a)



$$C_A(1 - z^{-1})V_o(z) = -C_3V_o(z) - C_2V_i(z) - C_1(1 - z^{-1})V_i(z)$$

Once a filter structure is obtained, its precise frequency response is determined through the use of discrete-time analysis

First-order Switched-capacitor Filter

$$C_A(1 - z^{-1})V_o(z) = -C_3V_o(z) - C_2V_i(z) - C_1(1 - z^{-1})V_i(z)$$

$$H(z) \equiv \frac{V_o(z)}{V_i(z)} = -\frac{\left(\frac{C_1}{C_A}\right)(1 - z^{-1}) + \left(\frac{C_2}{C_A}\right)}{1 - z^{-1} + \frac{C_3}{C_A}}$$

$$= -\frac{\left(\frac{C_1 + C_2}{C_A}\right)z - \frac{C_1}{C_A}}{\left(1 + \frac{C_3}{C_A}\right)z - 1}$$

$$z_p = \frac{C_A}{C_A + C_3} \qquad z_z = \frac{C_1}{C_1 + C_2}$$

$$H(1) = \frac{-C_2}{C_3}$$

Frequency response of the first-order filter

$$H(z) \equiv \frac{V_o(z)}{V_i(z)} = -\frac{\left(\frac{C_1}{C_A}\right)(1-z^{-1}) + \left(\frac{C_2}{C_A}\right)}{1 - z^{-1} + \frac{C_3}{C_A}} = -\frac{\left(\frac{C_1}{C_A}\right)(z^{1/2}-z^{-1/2}) + \left(\frac{C_2}{C_A}\right) z^{1/2}}{z^{1/2} - z^{-1/2} + \frac{C_3}{C_A} z^{1/2}}$$

Using: $z^{1/2} = \cos\left(\frac{\omega T}{2}\right) + j\sin\left(\frac{\omega T}{2}\right)$ & $z^{-1/2} = \cos\left(\frac{\omega T}{2}\right) - j\sin\left(\frac{\omega T}{2}\right)$

$$H(e^{j\omega T}) \equiv \frac{V_o(e^{j\omega T})}{V_i(e^{j\omega T})} = -\frac{j\frac{2C_1 + C_2}{C_A}\sin\left(\frac{\omega T}{2}\right) + \frac{C_2}{C_A}\cos\left(\frac{\omega T}{2}\right)}{j\left(2 + \frac{C_3}{C_A}\right)\sin\left(\frac{\omega T}{2}\right) + \frac{C_3}{C_A}\cos\left(\frac{\omega T}{2}\right)}$$

$\omega T << 1 \Rightarrow \omega << 1/T$:

$$= -\frac{j\left(\frac{C_1 + C_2/2}{C_A}\right)\omega T + \frac{C_2}{C_A}}{j\left(1 + \frac{C_3}{2C_A}\right)\omega T + \frac{C_3}{C_A}}$$

$$\left\{ \begin{array}{l} j\omega_z T = \dfrac{-C_2/C_1}{1 + \dfrac{C_2}{2C_1}} \\[2ex] j\omega_p T = \dfrac{-C_3/C_A}{1 + \dfrac{C_3}{2C_A}} \end{array} \right.$$

Switch Sharing

Fig. 14.18(a)

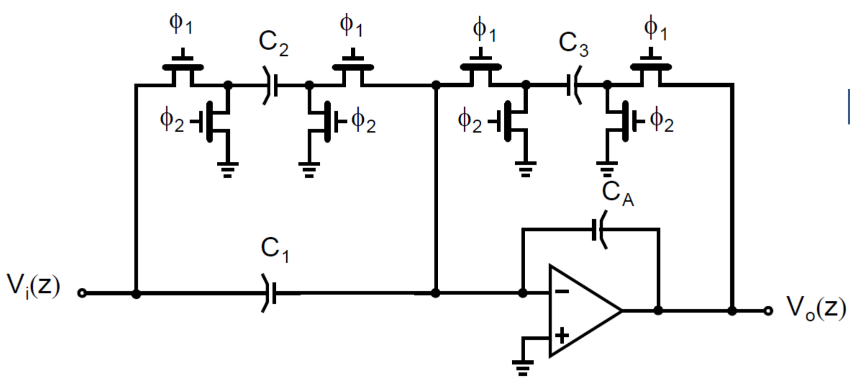
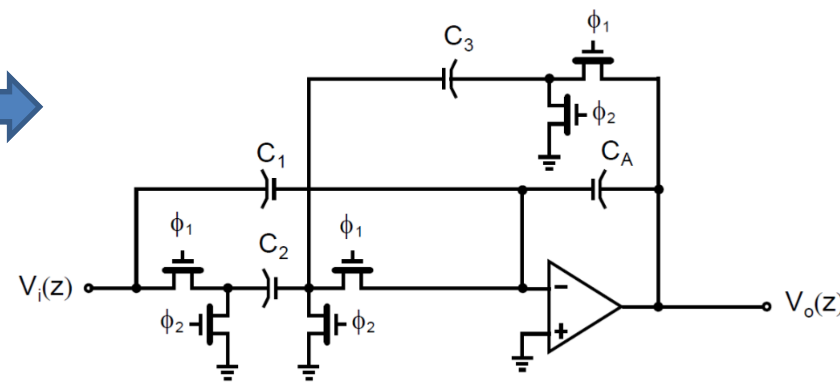
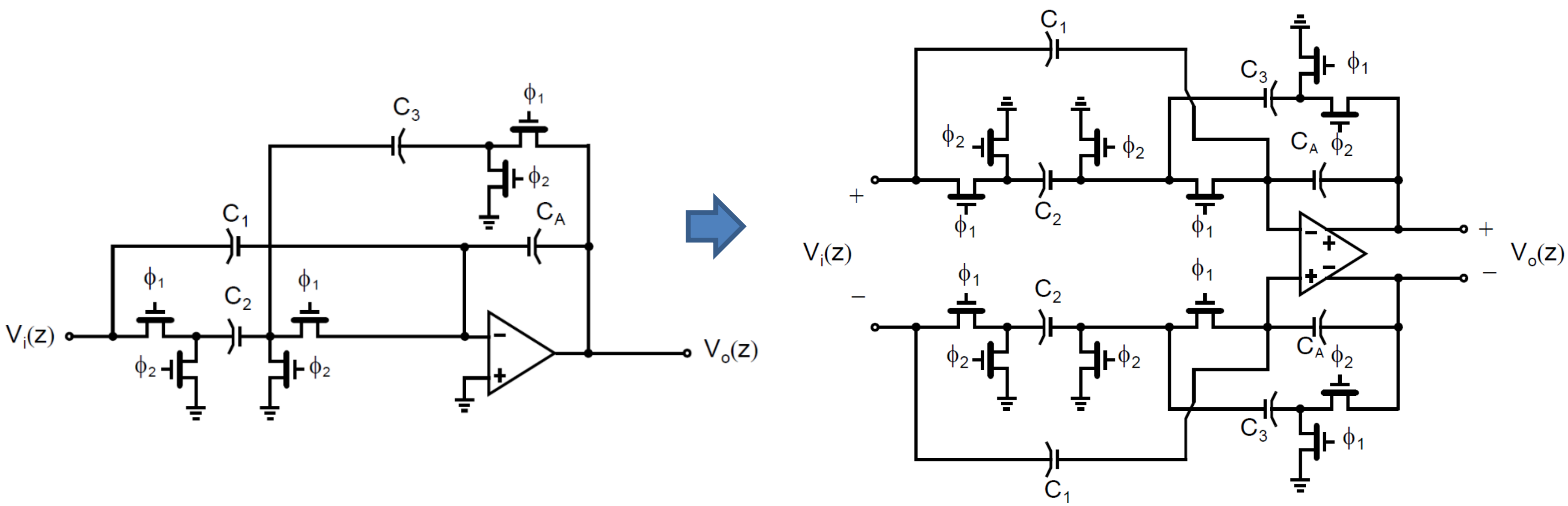


Fig. 14.19



Fully Differential Implementation



Allows for signal inversion by crossing wires

Biquad Filters

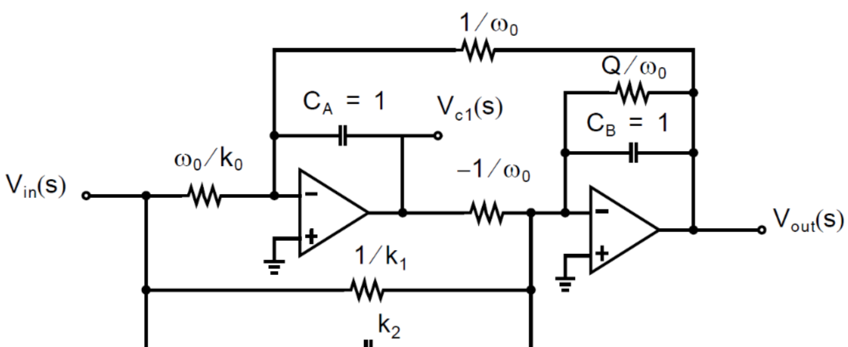
Secton 14.5

Discrete-Time Biquad Realizations

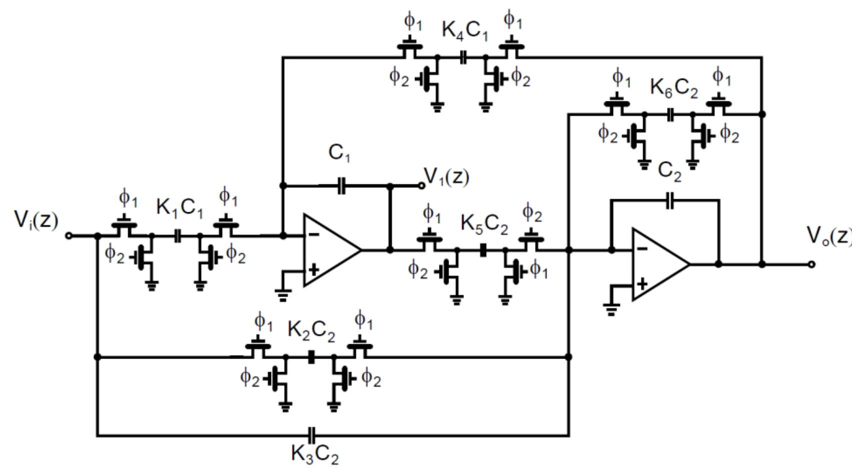
- Similar to continuous-time biquads, discrete-time biquads may be realized using two integrators in a negative feedback loop, one of the integrators having loss to provide stability
- Several different discrete-time biquads can be realized by varying the type (i.e., delayed, delay-free) and exact connection of switched-capacitor branches within the two-integrator loop
- The choice of which biquad to use depends upon the specific pole and zero locations sought.

Biquad #1

Active RC Prototype

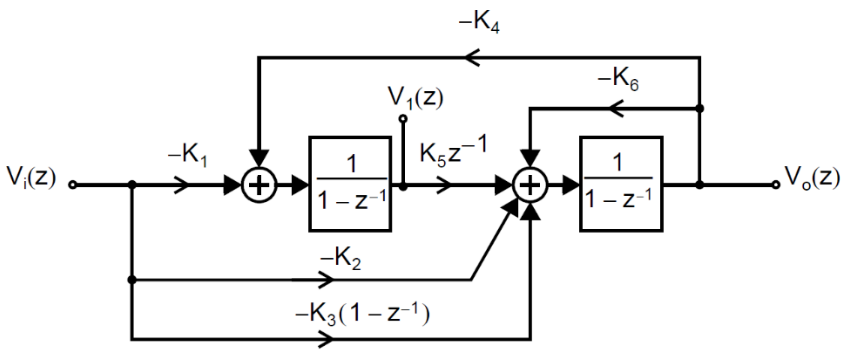


Switched-capacitor Equivalent

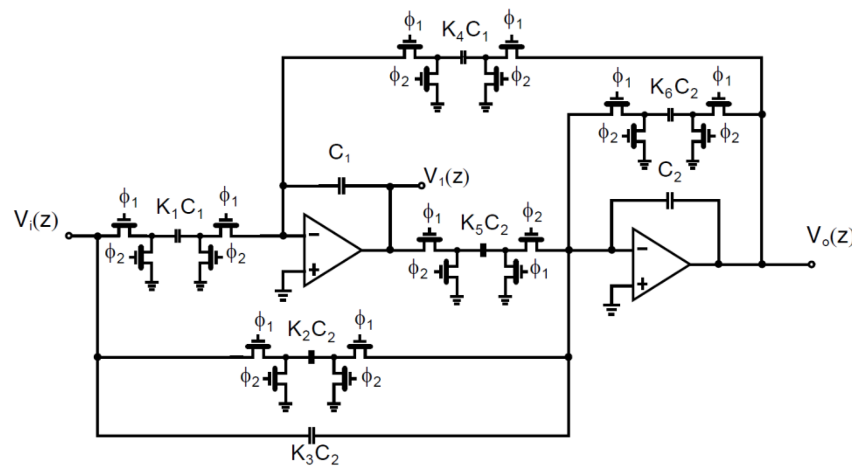


Biquad #1

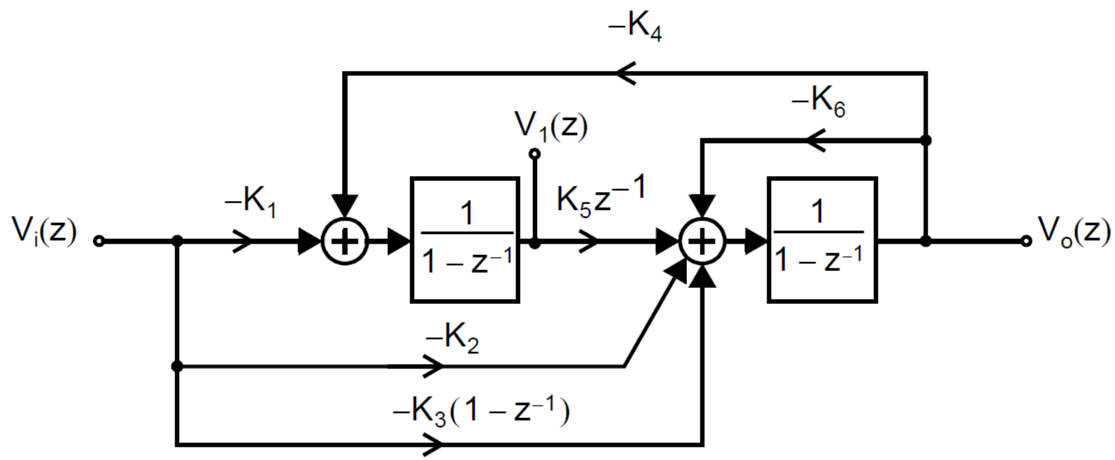
Signal Flow Graph



Switched-capacitor Equivalent



Biquad #1 Transfer Function



$$H(z) \equiv \frac{V_o(z)}{V_i(z)} = -\frac{(K_2+K_3)z^2+(K_1K_5-K_2-2K_3)z+K_3}{(1+K_6)z^2+(K_4K_5-K_6-2)z+1}$$

Biquad #1 Design Equations

$$H(z) = -\frac{a_2 z^2 + a_1 z + a_0}{b_2 z^2 + b_1 z + 1} = -\frac{(K_2 + K_3)z^2 + (K_1 K_5 - K_2 - 2K_3)z + K_3}{(1 + K_6)z^2 + (K_4 K_5 - K_6 - 2)z + 1}$$

$$K_3 = a_0$$

$$K_2 = a_2 - a_0$$

$$K_1 K_5 = a_0 + a_1 + a_2$$

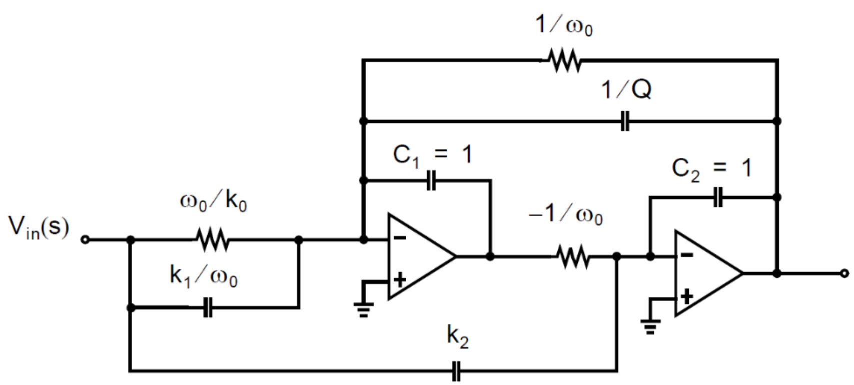
$$K_6 = b_2 - 1$$

$$K_4 K_5 = b_1 + b_2 + 1$$

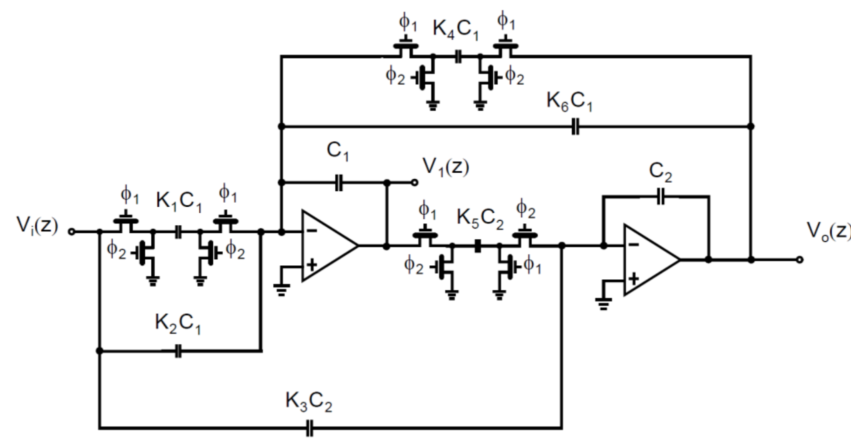
Extra degree of freedom in choice of K_5 determines the amplitude of the signal at V_1.

Biquad #2

Active RC Prototype



Switched-capacitor Equivalent



Biquad #2 Design Equations

$$H(z) = -\frac{a_2 z^2 + a_1 z + a_0}{z^2 + b_1 z + b_0} = -\frac{K_3 z^2 + (K_1K_5 + K_2K_5 - 2K_3)z + (K_3 - K_2K_5)}{z^2 + (K_4K_5 + K_5K_6 - 2)z + (1 - K_5K_6)}$$

$$K_1K_5 = a_0 + a_1 + a_2$$

$$K_2K_5 = a_2 - a_0$$

$$K_3 = a_2$$

$$K_4K_5 = 1 + b_0 + b_1$$

$$K_5K_6 = 1 - b_0$$

Extra degree of freedom in choice of K_5 determines the amplitude of the signal at V_1.